Extension of the Microwave Emission Model of Layered Snowpacks to Coarse-Grained Snow

Christian Mätzler* and Andreas Wiesmann *

The microwave emission model of layered snowpacks (MEMLS) is a multilayer and multiple-scattering radiative transfer model developed for dry winter snow using an empirical parametrization of the scattering coefficient (see the companion article). A limitation is in the applicable range of frequencies and correlation lengths. In order to extend the model, a physical determination of the volume-scattering coefficients, describing the coupling between the six fluxes, is developed here, based on the improved Born approximation. An exponential spatial autocorrelation function was selected. With this addition, MEMLS obtains a complete physical basis. The extended model is void of free parameters. The validation was done with two types of experiments made at the alpine test site, Weissfluhjoch: 1) radiometry at 11 GHz, 21 GHz, 35 GHz, 48 GHz, and 94 GHz of winter snow samples on a blackbody and on a metal plate, respectively, and 2) radiometric monitoring at 4.9 GHz, 10.4 GHz, 21 GHz, 35 GHz, and 94 GHz of coarse-grained crusts growing and decaying during melt-and-refreeze cycles. Digitized snow sections were used to measure snow structure in both experiments. The coarsest grains were found in the refrozen crusts with a correlation length up to 0.71 mm; the winter snow samples had smaller values, from 0.035 mm for new snow to about 0.33 mm for depth hoar. Good results have been obtained in all cases studied so far. ©Elsevier Science Inc., 1999

INTRODUCTION

A microwave emission model of layered snowpacks (MEMLS) was developed for dry winter snow by Wiesmann and Mätzler (1999) (the companion article to this article) within an ESA study (Pulliainen et al., 1998; see also Wiesmann et al., 1999b). It is a multilayer and multiple-scattering radiative transfer model applicable to the refractive snow medium, using six-flux theory to describe volume scattering and absorption, including radiation trapping due to total internal reflection. The scattering coefficient was determined empirically from measured snow samples (Wiesmann et al., 1998a), and the absorption coefficient, the effective permittivity, refraction, and reflection at the plane layer interfaces were physically described. The empirical model for the scattering coefficient (Wiesmann et al., 1998a) has a similarity to Rayleigh scattering, however, with a weaker frequency dependence. The scattering increases by a power law of the microwave frequency times the correlation length with a power of approximately 2.5. Above a certain frequency or above a certain correlation length, the increase will saturate in a similar way as Mie scattering does for spheres. The snow-sample experiments of Wiesmann et al. (1998a) were made with winter snow without very large grains (correlation length 0.035 mm to 0.33 mm). Thus the saturation effect was not observed, and thus was not included in MEMLS. Clusters of large grains form by melt metamorphism (Colbeck, 1986). Hard crusts of refrozen snow are formed when these clusters refreeze. If the model is applied to such crusts (Reber et al., 1987), the simulated brightness temperatures become too low at millimeter wavelengths. As an example, at 35 GHz the brightness temperature is underestimated by more than 30 K for a refrozen crust observed on 14 June 1984 with a correlation length of 0.71 mm. The error is even larger at 94 GHz. Therefore, a better model
is needed for the scattering coefficient, and anisotropic phase functions must be considered. A suitable model to be applied is one which describes the snow by statistical properties without the need for defining a grain size. Such a model is given by the improved Born approximation (Mätzler, 1998) where the snow structure is described by the spatial two-point correlation. Although characteristic autocorrelation functions are obtained for different grain shapes (Mätzler, 1997), exponential functions are assumed here for simplicity being in acceptable agreement with many observations and with theory (Debye et al., 1957).

MICROWAVE SCATTERING ACCORDING TO THE IMPROVED BORN APPROXIMATION

In the improved Born approximation, the bistatic volume-scattering coefficient (bistatic scattering cross section per unit volume, also the phase function if divided by the extinction coefficient, $\gamma_{\text{ext}}$) for scattering from direction $\mathbf{i}$ to $\mathbf{6}$ of isotropically oriented, freely arranged ice particles in air is given by (Mätzler, 1998):

$$\gamma^b(\mathbf{6}, \mathbf{i}) = \epsilon(1 - \epsilon) (\epsilon - 1)^2 K^2 \frac{\sin^2 \chi}{\kappa^3},$$  (1)

where $\chi$ is the polarization angle (between the incident electric field and the direction $\mathbf{6}$ of the scattered radiation), $\epsilon$ is the volume fraction of grains, $\epsilon$ is the ice permittivity, $\kappa$ is the vacuum wave number, $K$ is the ratio of the mean-squared electric fields inside and outside of the particles. This ratio depends on the shape of the ice particles. An analytic expression for $K^2$ was found for the case of ellipsoids:

$$K^2 = \frac{1}{3} \sum_{j \neq i} \left( \frac{\epsilon_j}{\epsilon_i + (\epsilon - 1) A_j} \right)^2,$$  (2)

Here $A_j$ is the depolarization factor of principal axis $j$ of the ellipsoidial grains, and $\epsilon_i$ is the apparent permittivity (Sihvola and Kong, 1988), given by

$$\epsilon_i = \epsilon_{\text{eff}} (1 - A_i) + A_i.$$  (3)

For the effective permittivity $\epsilon_{\text{eff}}$ the effective medium formula of Polder and van Santen (1946) can be used [Eq. (22) in Mätzler (1998)]. Measurements of the effective permittivity of dry snow made with high accuracy near 1 GHz showed that the effective medium theory, using Eqs. (2) and (3), provides a good quantitative description of natural snow (Mätzler, 1996). The factor $I$ in Eq. (1) is a Fourier integral of the autocorrelation function. This function was determined experimentally for all snow experiments to be presented here. An exponential function, $\exp(-x/p_{\text{sc}})$, gave a good fit in most cases. The correlation length $p_{\text{sc}}$ is the only parameter. For this function, the factor $I$ is given by Eq. (4),

$$I = \frac{1}{a_{\text{sc}}^7} \exp(-x/p_{\text{sc}}) x \sin(ax) \, dx = \frac{2p_{\text{sc}}^3}{(1 + a^2 p_{\text{sc}}^2)^{3/2}},$$  (4)

where $a$ is the scattering parameter, defined by Eq. (5),

$$a^2 = 4\epsilon_{\text{ir}} k^2 \sin^2(\theta/2) = 2\epsilon_{\text{ir}} k^2 (1 - \cos \theta)$$  (5)

and $\theta$ is the scattering angle. The correlation length $p_{\text{sc}}$ was determined from the $e$-folding distance of the exponential fitted to the measured correlation function.

The analysis of the directional behavior of Eq. (1) gives the $\sin^2 \chi$ dipole pattern at low frequency ($a \ll 1$). With increasing $k p_{\text{sc}}$, the scattering is more and more enhanced in the forward direction, whereas saturation occurs for backscattering. This situation occurs for coarse-grained snow at millimeter wavelengths.

THE FIELD FACTOR $K^2$

According to the effective medium theory, the factor $K^2$ of Eq. (1) depends on grain shape and snow density. The shape dependence is relatively weak; therefore, the real situation can be well modeled with idealized particles. Two extreme situations may be of special interest: the measured permittivity of dry snow, and spherical particles. According to the dielectric data of Mätzler (1996), dry snow can be described by oblate spheroidal ice particles whose axial ratio is a function of the ice-volume fraction $\epsilon$. Then $A_j = A_2$, and $A_1 = 1 - 2A_2$. The following dependence is a fit for oblate grains for $\epsilon < 0.6$:

$$A_1 = \begin{cases} 
0.1 + 0.5\epsilon, & 0 < \epsilon < 0.33, \\
0.476 - 0.64\epsilon, & 0.33 < \epsilon < 0.6. 
\end{cases}$$  (6)

Using this depolarization factor in (2) and (3) and the effective permittivity of dry snow according to Mätzler (1996), we get the results shown by the solid line of Figure 1. On the other hand, spherical ice particles have $A_j = 1/3$, resulting in a $K^2$ factor shown as the dashed line in Figure 1; its values are smaller than those of the non-spherical snow particles. The two lines approach each other near $\epsilon = 0.33$, where $A_1$ of Eq. (6) has a maximum near 1/3.

THE SIX-FLUX COEFFICIENTS

The six-flux theory was originally described by Emslie and Aronson (1973). An important concept introduced by Wiesmann et al. (1998a) is the division between the vertical and horizontal fluxes by the critical angle of total reflection. The radiation is divided into six beams: 1) an upward and 2) a downward beam, defined by the escapable radiation, that is, whose direction cosine $\mu$ with the vertical axis obeys Eq. (7):

$$|\mu| > \mu_c = \cos \theta_c = \sqrt{1 - (\cos \alpha')^{-2}},$$  (7)

where $\theta_c$ is the critical angle for total reflection and $\alpha'$ is the real part of the refractive index of the medium, and (three to six) four “horizontal” beams of trapped radiation (with $|\mu| < \mu_c$). Averaging of $\gamma^b(\mathbf{6}, \mathbf{i})$ over all scattering directions leads to the scattering coefficient (Ishimaru, 1978) in Eq. (8):
The mean-squared electric field ratio $K^2$ versus ice-volume fraction $v$ for oblong ice particles found in dry snow in the range, $0.05 \leq v \leq 0.5$ (upper line), according to Mätzler (1996) and for spherical ice particles (lower line).

$$\gamma_b = \frac{1}{4\pi} \int \int \int \gamma_{b}(\mu, \varphi, \mu', \varphi') \, d\Omega_{\mu} \, \frac{1}{2} \int \Gamma(\mu, \varphi) \, d\mu$$

(8)

where $\mu, \varphi$ are the direction cosines and azimuths of the incident ($i$) and scattered ($o$) directions, respectively, and $\Gamma$ is the azimuth-averaged $\gamma_{b}$.

$$\Gamma(\mu, \varphi) = \frac{1}{2\pi} \int \gamma_{b}(\mu, \varphi, \mu', \varphi') \, d\varphi'$$

(9)

After division by the extinction coefficient, this function becomes the azimuth-averaged phase function. Now, in order to get the coefficients needed in MEMLS to describe the coupling between the six streams, we consider incident radiation from direction $(\mu, \varphi)$; the scattering cross-sections per unit volume for scattering in the backward beam $\gamma_{0b}$ in one of the four transverse (cross) beams $\gamma_{0i}$, and in the forward beam $\gamma_{0f}$, respectively, are integrals over the respective $\mu_i$ intervals:

$$\gamma_{0b}(\mu_i) = \frac{1}{2} \int \Gamma(\mu_i, \varphi) \, d\mu_i$$

$$\gamma_{0d}(\mu_i) = \frac{1}{2} \int \Gamma(\mu_i, \varphi) \, d\mu_i$$

$$\gamma_{0f}(\mu_i) = \frac{1}{2} \int \Gamma(\mu_i, \varphi) \, d\mu_i$$

(10)

If the incident radiation is diffuse, the six-flux coefficients become average values of the Eqs. (10):

$$\gamma_{0} = \frac{1}{2} \gamma_{0b}(\mu_i) \, d\mu_i$$

(11)

The summation of these coefficients leads to the scattering coefficient, again:

$$\gamma_{0} + 4\gamma_{1} + \gamma_{2} = \gamma_{s}$$

(12)

Equations (11) and (12) replace Eqs. (29), (30), and (59) of the companion article (Wiesmann and Mätzler, 1999).

In a dense medium the widths of the forward and backward beams are not very wide. Therefore, Eqs. (11) are useful approximations for the six-flux coefficients also for directed radiation. For $\mu_i=0$ the transverse fluxes disappear, and the six-flux model degenerates to the simpler two-flux model, described, for instance, by Ishimaru (1978) in his Chapter 10. In this case, the main coefficient of interest is $\gamma_{0}$, given by Eq. (11a):

$$\gamma_{0} = \frac{1}{2} \gamma_{0b}(\mu_i) \, d\mu_i$$

(11a)

A formal difference, that is, the missing factor 1/2 in Ishimaru’s expression for the two-flux scattering coefficient S (10A-8) is explained by the effective direction cosine $\mu_{eff}$ for up- and down-propagating radiation. Ishimaru’s coefficient is related to a change in the opacity with respect to a change in vertical direction, whereas our coefficients are related to the change of scattering cross-section with respect to a change in effective path length. The ratio, vertical direction to effective path length, just defines $\mu_{eff}$; the ratio is 1/2 in the two-flux model with isotropic radiation. For the case of the six-flux model, the effective direction cosine is given in Eq. (13):

$$\mu_{eff} = \frac{1}{2} \int \mu \, d\mu = \frac{1}{2} - \frac{\mu_i^2}{2}$$

(13)

which becomes 1/2 for $\mu_i=0$ (see MEMLS Documentation, Wiesmann and Mätzler, 1998).

The absorption coefficient $\gamma_{a}$ in the improved Born approximation is given in Eq. (14),

$$\gamma_{a} = v \kappa \gamma_{s} K^2$$

(14)

and finally, the extinction coefficient is the sum given in Eq. (15),
VALIDATION WITH REFROZEN CRUSTS

Melt metamorphism in wet snowpacks makes the larger grains grow while the smaller ones disappear (Colbeck, 1986). The process is very rapid during snow melt, changing a fresh snowpack to firnlike snow within a few days. When such snow refreezes, the microwave emissivity can be unusually low, reaching values between 0.4 and 0.5 at frequencies of 35 GHz and 94 GHz (e.g., Reber et al., 1987; Mätzler, 1994). Refreezing mainly occurs during clear nights, forming a crust at the surface with a continuously increasing thickness. The crust is easily recognized by its hardness, and thus can be measured manually. Radiometric data of growing crusts are very useful to validate the snowpack emission model because the thickness response and the spectral response can be modeled and measured simultaneously. The emission is fully determined by the properties of the refrozen snow, whereas the wet snow below the crust acts nearly as a blackbody at 0°C. Reber et al. (1987) used the conventional Born approximation to simulate optically thin backscattering of crusts formed during freeze–melt events in May and June 1984 at the test site, Weissfluhjoch, Davos, Switzerland. Here we use the same crust data, but now applied to MEMLS, comparing the results with measured brightness temperatures. Atmospheric effects were included by taking the measured sky radiation as input to the model. Figures 4 and 5 show crust thickness, observed and modeled brightness temperatures at h- and v-polarization versus time. Each graph represents data at one frequency at an incidence angle of 50°. The liquid-water content was assumed to be 2% by volume anywhere below the crust. Crusts are always dry, and their temperature was at or below 0°C. Liquid-water content appeared in the surface layer during the melt phase, thus transforming the crust back to wet snow again within a few hours. Table 2 defines the snow state and model input data of early May, and Table 3 shows the simpler, but coarse-grained situation of June. Since the wet snow hardly influences the observable radiation, the zero point of the vertical z-axis was not chosen to be at the ground; instead it was taken near the bottom of the deepest crust.

The freeze–melt–freeze event of 8–10 May 1984 of Figure 4 represents an early stage in the melt season. During two consecutive nights, the crust grew from the surface to a depth of more than 20 cm (shown as negative value). Strong stratification was observed. Especially up to about 10 ice lenses were identified within the top 25 cm of the pack. Furthermore, a 3–4 cm thick layer of fresh snow accumulated on 8 May. During the following day, the fresh snow suffered melt metamorphism;

Table 1. Data of Snow Samples 9 and 12 Used to Compute the Spectra of Figure 2

<table>
<thead>
<tr>
<th>Sample No.</th>
<th>Thickness (cm)</th>
<th>Density (g/cm³)</th>
<th>Temperature (K)</th>
<th>ρₑ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>8.2</td>
<td>0.367</td>
<td>269.3</td>
<td>0.056</td>
</tr>
<tr>
<td>12</td>
<td>9.5</td>
<td>0.335</td>
<td>273.0</td>
<td>0.178</td>
</tr>
</tbody>
</table>

\[ \gamma = \gamma_0 + \gamma_1 \]

For the numerical integration, expressions for the scattering and polarization angles in terms of the direction cosines are needed. The following relations [Eqs. (16), (17a), and (17b)] apply, choosing \( \varphi_i = 0 \):

\[ \cos \theta = \mu \cos \varphi_0 + \sqrt{1 - \mu^2 (1 - \mu^2)} \cos \phi_0 \], \quad (16) \]

\[ \cos^2 \chi = (1 - \mu^2) \sin^2 \phi_0 \], \quad h polarization, \quad (17a) \]

\[ \cos^2 \chi = \left( \mu \sqrt{1 - \mu^2} - \mu_1 \sqrt{1 - \mu_1^2} \cos \phi_0 \right)^2 \], \quad v polarization. \quad (17b) \]

A simplification is made at this point, based on the observation that the scattering coefficients are independent of polarization (Wiesmann et al., 1998a). Also the difference between Eqs. (17a) and (17b) is not large after integration over azimuth (9). Therefore, we use the average value of (17a) and (17b), thus dropping the polarization dependence; the result is Eq. (17c):

\[ \cos^2 \chi = 1 - 0.5 \sin^2 \theta \]. \quad (17c) \]

The averaging has a physical basis, in that the plane of incidence is changed after each scatter event. The polarization effects considered are those arising at the interface between adjacent layers (Wiesmann and Mätzler, 1999).

VALIDATION USING WINTER-SNOW SAMPLES

As a first test of the described scattering model, the snow-sample data of Wiesmann et al. (1998a) were used. The correlation lengths were determined by fitting the measured correlation functions to exponentials (Reber et al. 1987; Weise, 1996). The input data of a fine-grained (Sample 9) and a medium-grained snow sample (Sample 12) are given in Table 1, and computed and measured emissivities of these samples are plotted versus frequency in Figure 2.

Results of other snow samples are similar; furthermore, they all compare well with the MEMLS simulation using the empirical fits, Eq. (59) in the companion article for the scattering coefficient. This is shown by the 94 GHz data of Figure 3, comparing emissivities of the 20 samples computed in the two ways. Similar results are obtained at lower frequencies, however, with increasing concentration of data points near the (0,0) and (1,1) corners.
Figure 2. Emissivity spectra of snow Samples 9 (left) and 12 (right) situated on a metal plate (lower values) and on a blackbody (higher values), at h- and v-polarization, incidence angle $=45^\circ$, modeled with MEMLS using the improved Born approximation (solid lines, v-pol.), (dashed lines h-pol.), and compared with measurements (h, v, H, V).

thus stronger scattering appeared during the following night. The metamorphism of the top layer was simulated by a growing correlation length during the wet phase on 9 May. Below the fresh snow, five ice lenses were taken into account by the simulation (Table 2). The effect of these lenses is to increase the polarization difference with increasing number of lenses within the growing crust, especially at frequencies where the lens thickness is close to a quarter wavelength. This effect is most apparent at 21 GHz.

The freeze–melt event of 13–14 June 1984 (Fig. 5) was observed at a late stage when melt metamorphism

Figure 3. Comparison of computed emissivities at 94 GHz of snow samples, above a metal plate (points h-pol., diamonds v-pol.) and above a blackbody (circles h-pol., triangles v-pol) according to scattering coefficients using the empirical fit, Eq. (59) of the companion article, versus the improved Born approximation. Maximum and minimum correlation length are 0.035 mm and 0.327 mm, respectively.
Figure 4. Freeze–melt–freeze event of 8–10 May 1984 at Weissfluhjoch, showing the time variation of crust depth $d$ and of brightness temperatures for $\theta=50^\circ$ at 21 GHz, 35 GHz, and 94 GHz, measurements (pointed lines) and simulations with the improved Born approximation (simple lines), dashed lines for data at h-pol., solid lines at v-pol.
Figure 5. Freeze–melt event of 13–14 June 1984 at Weissfluhjoch, showing the time variation of crust depth $d$ (diamonds) and of brightness temperatures for $\theta=50^\circ$ at 4.9 GHz, 35 GHz, and 94 GHz. Measurements (pointed lines) and simulations with the improved Born approximation (simple lines), dashed lines for data at h-pol., solid lines at v-pol.
was strongly advanced. The crust reached a maximum thickness of 7 cm. The snow grains were extremely large. The fit to an exponential correlation function indicated a correlation length of 0.71 mm. A thin surface layer of finer-grained snow at lower density, and a high-density layer below 7 cm depth was indicated by the in situ observations (Krusti, 1985/6).

DISCUSSION

The modeled emissivities of the winter snow samples (Figs. 2 and 3) show good agreement with the measurements over the observed spectrum from 11 GHz to 94 GHz. The scattering parameter $\alpha$ was found to be $\alpha < 1.6$ for all samples at 94 GHz. This explains why a simple empirical expression can be used for winter snow. On the other hand, the data of the refrozen crusts (Figs. 4 and 5) cannot be reproduced from the empirical formulae of the scattering coefficient presented in the companion article. Therefore, the simulation was limited to the improved Born approximation. In both examples the modeled data compare reasonably well with the measurements not only as a function of frequency, but also versus crust thickness. Strong volume scattering appears by the significantly depressed brightness temperatures at 35 GHz and 94 GHz, v- and h-polarization, reaching similar values in both crusts, and significant scattering is also apparent at 21 GHz near the time of maximum crust thickness. The nonlinear decrease of the brightness temperature at 94 GHz with respect crust thickness is an indication of saturation due to high optical thickness and limited penetration depth. This saturation effect is not apparent at the lower frequencies. The effect of the ice lenses in May is shown by the increasing polarization difference (especially at 21 GHz) during the growth of the crust. In the data of June (Fig. 5) the polarization difference remains small during the crust formation because ice lenses were absent. At the lower frequencies (4.9 GHz and 10.4 GHz), volume scattering is almost invisible; however, a decreased emissivity is observed at h-polarization during the melt periods as an indication of the increased liquid-water content. Only a few values of liquid-water content had actually been measured with dielectric snow probes (Mätzler, 1987). Quasiperiodic variations of the brightness temperature at 4.9 GHz and at 10.4 GHz (h-pol.) are apparent (see Fig. 5 at 4.9 GHz) during the refreezing phase. The effect is explained by a coherent superposition of reflections at the upper and lower crust surface. Since the model considers coherent effects only for thin layers (two-way phase differences up to 1.5$\pi$), these variations disappear after the first maximum. By averaging the data over larger footprints, these oscillations would also disappear from the observations.

CONCLUSIONS

The improved Born approximation with an exponential correlation function was used to compute six-flux scattering and absorption coefficients for the microwave emission model of layered snowpacks (MEMLS). In this way MEMLS obtained a full physical basis. Limited accuracy may still result from simplifying assumptions, such as the limitation of spatial directions to six streams. Furthermore, polarization-independent scattering coefficients are assumed. This simplification is supported by the experimental observations of Wiesmann et al. (1998a); it allows a scalar treatment of volume scattering, a faster computation than for a fully polarimetric formulation. Observations from various kinds of snowpacks were used to validate the model. It turned out that it is useful in all cases tested, clearly exceeding the range of the empirical fit. The good results at all frequencies, and up to the coarsest grains ever observed by our instruments, leads us to the expectation that the improved MEMLS may be applicable to the modeling of microwave emission from snowpacks all over the globe.

An advantage of the improved Born approximation versus discrete particle models is the simple (two-point) description of the snow structure. For an exponential correlation function, the correlation length is the only microscopic parameter, which can directly be measured from thin sections. Also the limited information needed

Table 2. Model Input Data of the Snowpack on 9 May 1984 (Fig. 4)

<table>
<thead>
<tr>
<th>$j$</th>
<th>$z_j (m)$</th>
<th>$\text{dens}_j (g/cm^3)$</th>
<th>$T_j (K)$</th>
<th>$\rho_{a_j} (mm)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0000</td>
<td>273.2</td>
<td>0.300</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.0890</td>
<td>0.40</td>
<td>273.2</td>
<td>0.240</td>
</tr>
<tr>
<td>3</td>
<td>0.0900</td>
<td>0.70</td>
<td>273.2</td>
<td>0.324</td>
</tr>
<tr>
<td>4</td>
<td>0.1060</td>
<td>0.40</td>
<td>273.2</td>
<td>0.240</td>
</tr>
<tr>
<td>5</td>
<td>0.1722</td>
<td>0.70</td>
<td>273.2</td>
<td>0.324</td>
</tr>
<tr>
<td>6</td>
<td>0.1220</td>
<td>0.40</td>
<td>273.2</td>
<td>0.240</td>
</tr>
<tr>
<td>7</td>
<td>0.1236</td>
<td>0.70</td>
<td>273.2</td>
<td>0.240</td>
</tr>
<tr>
<td>8</td>
<td>0.1470</td>
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<td>0.240</td>
</tr>
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<td>273.2</td>
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</tr>
<tr>
<td>10</td>
<td>0.1690</td>
<td>0.38</td>
<td>273.2</td>
<td>0.240</td>
</tr>
<tr>
<td>11</td>
<td>0.1710</td>
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<td>273.2</td>
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</tr>
<tr>
<td>12</td>
<td>0.2000</td>
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<td>273.2</td>
<td>0.540</td>
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<tr>
<td>13 surface</td>
<td>0.2370</td>
<td>0.31</td>
<td>273.2</td>
<td>0.300</td>
</tr>
</tbody>
</table>

*Height $z_j$ of layer surface, density $\text{dens}_j$, temperature $T_j$, and correlation length $\rho_{a_j}$ of Layer $j$. The volumetric liquid water content (not shown) changed with time and layer position.

Table 3. Same as Table 2, but for the Snowpack of 13 June 1984 (Fig. 5)

<table>
<thead>
<tr>
<th>$j$</th>
<th>$z_j (m)$</th>
<th>$\text{dens}_j (g/cm^3)$</th>
<th>$T_j (K)$</th>
<th>$\rho_{a_j} (mm)$</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>0.0000</td>
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<td>0.300</td>
<td></td>
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<td>0.0500</td>
<td>0.65</td>
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<td>0.50</td>
</tr>
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<td>3</td>
<td>0.1200</td>
<td>0.44</td>
<td>273.2</td>
<td>0.75</td>
</tr>
<tr>
<td>4 surface</td>
<td>0.125</td>
<td>0.35</td>
<td>273.2</td>
<td>0.30</td>
</tr>
</tbody>
</table>
on grain shape, the $K^2$ factor, was obtained from measurements. On the other hand, if snow is modeled by discrete particles, the problem arises of how to derive these from the snow structure, a question which has not yet found a satisfactory answer, an ill-posed problem, indeed. Still, it is possible to determine the correlation function and its characteristic length from given particle shapes and sizes [see, e.g., Mätzler (1997) for freely arranged particles].

Concerning the error assessment and sensitivity analysis of the present model, the statements given in the discussion of the companion article (Wiesmann and Mätzler, 1999) apply here as well.

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