Microwave Emission Model of Layered Snowpacks

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A thermal microwave emission model of layered snowpacks (MEMLS) was developed for the frequency range 5–100 GHz. It is based on radiative transfer, using six-flux theory to describe multiple volume scattering and absorption, including radiation trapping due to total reflection and a combination of coherent and incoherent superpositions of reflections between layer interfaces. The scattering coefficient is determined empirically from measured snow samples, whereas the absorption coefficient, the effective permittivity, refraction, and reflection at layer interfaces are based on physical models and on measured ice dielectric properties. The number of layers is only limited by computer time and memory. A limitation of the empirical fits and thus of MEMLS is in the range of observed frequencies and correlation lengths (a measure of grain size). First model validation for dry winter snow was successful. An extension to larger grains is given in a companion article (Mätzler and Wiesmann, 1999). The objective of the present article is to describe and illustrate the model and to pave the way for further improvements. MEMLS has been coded in MATLAB. It forms part of a combined land-surface-atmosphere microwave emission model for radiometry from satellites (Pulliainen et al., 1998). ©Elsevier Science Inc., 1999

INTRODUCTION

The observation of passive microwave signatures of different types of snowpacks (e.g., Mätzler, 1994) and the need to simulate their microwave emission for future applications was the motivation for the development of a microwave emission model of layered snowpacks (MEMLS). Since the penetration depth of microwaves and the scattering coefficient depend on frequency and snow properties, a reasonable model should account for realistic snow profiles of the sensitive parameters, such as correlation length, density, and temperature, and in case of wet snow also the liquid-water content. A successful model might then be used for the development of remote sensing tools for observing processes on and within the snowpack. An important internal process, being responsible for the release of large avalanches, is temperature-gradient metamorphism, often taking place near the bottom of the snowpack. This process is based on large water-vapor gradients; it leads to the formation of cohesionless depth hoar (Armstrong, 1977; 1985). Microwave emission of depth-hoar layers are very special due to the enhanced volume-scattering by the large crystals (Weise, 1996). On the other hand, snowpack stratification has important implications for physical processes, such as avalanche formation by weak layers (Föhnt et al., 1998; Fierz, 1998), reduced vertical diffusion of water vapor and heat transfer, and percolation of liquid water in case of wet snow (Arons and Colbeck, 1995, and references therein). Snow layers also mark meteorological events in the history of snowpacks, and thus they contain information about past weather conditions. Specific microwave signatures related to stratification are expressed, for instance, by polarization features of microwave emission, and sometimes by special spectral properties. Since the earliest radiometer observations, interest has been paid to layering of polar ice sheets (Gurvich et al., 1973; Zhang et al., 1989; Rott et al., 1993; Surdyk and Fily, 1993; Steffen and Abdalati, 1999), and layer effects were also found in snow-covered sea ice (e.g., Mätzler et al., 1984). Microwave emission of layered media, such as firn, was computed in the past by several authors (e.g., Gurvich et al., 1973; Djermakoye and Kong, 1979; West et al., 1996). Their models are applicable at frequencies below about 20 GHz, where volume scattering by the granular snow structure is not dominant.
When both volume scattering by snow grains and by the stratification are relevant, the models become more complex. An example is the strong fluctuation theory (Stogryn, 1986). In the implementation by Surdyk (1992), this model was tested by the experimental snow data of Weise (1996) to be used also in the present work. The model validation clearly failed, probably because multiple scattering is ignored. Indeed, West et al. (1993) showed on the basis of the dense-medium radiative transfer theory applied to backscattering from snow that multiple scattering by snow grains is important.

Driven by the need for a realistic snowpack emission model, we started with the development of MEMLS, including multiple scattering both by stratification and by the granular snow structure, and a tuned combination of coherent and incoherent superpositions of different scattering contributions. First, the detailed behavior of single layers was investigated experimentally by Weise (1996) and later by Wiesmann et al. (1998). The work lead to a physical model of the emissivity of homogeneous snow slabs in the frequency range from 10 GHz to 100 GHz and for correlation lengths from 0.05 mm to 0.3 mm. With the exception of the empirical determination of the scattering coefficient, the parameters are physically based. In MEMLS this information is applied to describe the behavior of individual layers. Here, the snowcover is considered as a stack of n horizontal, planar layers, characterized by thickness, correlation length, density, liquid-water content, and temperature. The sandwich model of Wiesmann et al. (1998) is extended to couple internal scattering and reflections at the interfaces of all layers. Internal volume scattering is accounted for by a six-flux model (streams in all space directions). Homogeneity in the horizontal directions reduces to the well-known two-stream model (up- and downwelling streams) where the two-stream absorption and scattering coefficients are functions of the six-flux parameters. The absorption coefficient depends on density, frequency, and temperature, and the scattering coefficient depends on the correlation length, density, and frequency (Wiesmann et al., 1998).

The purpose of this article is to describe and illustrate the model and to pave the way for further improvements. Details on the realization are given by Wiesmann and Mättler (1998).

MICROWAVE EMISSION MODEL OF LAYERED SNOWPACKS

The snowcover is assumed to be a stack of n horizontal snow layers \((j=1, \ldots, n)\) with planar boundaries between air–snow and snow–snow (Fig. 1), characterized at frequency \(f\), polarization \(p\), and incidence angle \(\theta=\theta_n\) by

- the snowpack brightness temperature \(T_b\),
- the ground–snow interface reflectivity \(s_0\) and the ground temperature \(T_0\).

\[\begin{align*}
\text{Figure 1. A multilayer system with a wave incident from above at an angle } \theta_n. \\
& s_n \quad T_n \quad t_n \quad r_n \quad e_n \\
& s_{n-1} \quad T_{n-1} \quad t_{n-1} \quad r_{n-1} \quad e_{n-1}
\end{align*}\]

\[\begin{align*}
& d_n \quad T_n \quad t_n \quad r_n \quad e_n \\
& d_{n-1} \quad T_{n-1} \quad t_{n-1} \quad r_{n-1} \quad e_{n-1}
\end{align*}\]

- the interface reflectivity \(s_j\) on top of each layer \(j\),
- the thickness \(d_j\), temperature \(T_j\), internal reflectivity \(r_j\), emissivity \(e_j\), and transmissivity \(t_j\) of each layer,
- the downwelling (sky) radiation, expressed by the brightness temperature \(T_{sky}\).

The snow-ground interface is allowed to be rough; we only specify its reflectivity \(s_0\) based on observation or theory.

Radiation in Multilayered Media

Considering layer \(j\) in Figure 2, we can derive the following equations relating the up- and downwelling brightness temperatures at the layer boundaries:

\[A_j=r_jB_j+t_jC_j+e_jT_j,\]

\[B_j=s_{j-1}A_j+(1-s_{j-1})D_{j-1},\]

\[\text{Figure 2. Vertical fluxes at the boundaries of layer } j.\]
In order to compute realistic values of $r_j$, the Six-Flux Model identifies as the ground temperature
\[ D_n = T_b, \]
and $s_j$ is the ground-snow reflectivity. Similarly, for $j = n$, we have in Eq. (3) a term with $A_{n+1}$, namely, the sky brightness
\[ A_{n+1} = T_{d_{n+1}}. \]
Solving the system of Eqs. (1)–(4) for $j = 1$ to $n$, and using Eqs. (5) and (6) leads to $D_n$ and thus to the brightness temperature $B_{n+1} = T_b$ of the snowpack according to
\[ T_b = (1-s_n)D_n + s_n T_{d_{n+1}}. \]
Sometimes the emissivity $e_{sp}$ or the transmissivity $t_{sp}$ of the whole snowpack is of more interest than the brightness temperature $T_b$. The computation of these parameters is not obtained by simply dividing $T_b$ by a physical temperature because the snowpack is not regarded as isothermal. Nevertheless, by computing $T_b$ under various illumination conditions, the required quantities can be found. As an example, we can get the total emissivity (snowpack and background) from one minus the reflected fraction of the downwelling sky radiation by choosing two choices of $T_{d_{n+1}}$ (0 K and 100 K) in Eq. (8):
\[ e_{sp} = 1 - \frac{T_b(T_{d_{n+1}} = 100 \text{ K}) - T_b(T_{d_{n+1}} = 0 \text{ K})}{100 \text{ K}}. \]
If only one layer is present, $T_b$ follows from Eq. (7), where $D_n = D_1$ is given by Eq. (10):
\[ D_1 = \left[ \frac{r_1(1 - s_1)T_0 + r_1(1 - s_1)T_{d_2} + e_1T_1}{1 - r_1s_1} + r_1(1 - s_1)T_{d_2} + e_1T_1 \right] \left[ 1 - r_1s_1 \right]^{-1}. \]

**Figure 3.** The six fluxes within a selected layer $j$.

\[ \text{Radiative Transfer in a Refractive Medium—the Six-Flux Model} \]

In order to compute realistic values of $r_j$, $e_j$, and $t_j$, volume scattering and absorption within each layer $j$ are to be described by a radiative transfer model. An important distinction between radiative transfer in the atmosphere and in snow is that refraction plays a role in snowpacks. Even more important is the fact that total internal reflection occurs for certain rays. Our model takes these processes into account. We use a simplified radiative transfer model, reducing the radiation at a given polarization to six fluxes streaming along and opposed to the three principal axes of the slab (Fig. 3). The horizontal fluxes represent trapped radiation, that is, radiation whose internal incidence angle $\theta$ is larger than the critical angle $\theta_c$ for total reflection:
\[ \theta > \theta_c = \arcsin \left( \frac{1}{n'} \right), \]
where $n'$ is the real part of the refractive index of the slab. Only the vertical fluxes ($\theta = 0$) or the transmissivity $t_{sp}$ can be obtained from Eq. (9):
\[ t_{sp} = \frac{T_b(T_b = 273 \text{ K}) - T_b(T_b = 173 \text{ K})}{100 \text{ K}(1 - s_n)}. \]

If only one layer is present, $T_{d_{n+1}}$ is obtained from Eqs. (12) and (13) leading to two-flux equations [Eqs. (15) and (16)] with modified coefficients:
The reflectivity coefficient (Wiesmann et al., 1998). For constant $T$ and constant coefficients within the given layer, $T_{0j}$ and $T_{2j}$ can be written as Eq. (19):

$$
T_{0j} = T + A \exp(\gamma d') + B \exp(-\gamma d'),
$$
(19)

$$
T_{2j} = T + r_0 A \exp(\gamma d') + r_0 B \exp(-\gamma d'),
$$
(20)

where [Eq. (21)]

$$
d' = d/\cos \theta.
$$
(21)

The reflectivity $r_0$ at infinite layer thickness is given by Eq. (22),

$$
r_0 = \gamma_0 (\gamma_0' + \gamma_0 + \gamma)^{-1},
$$
(22)

and the damping coefficient $\gamma$ (eigenvalue) is given by Eq. (23),

$$
\gamma = \sqrt{\gamma_0' (\gamma_0' + 2\gamma_0)}.
$$
(23)

For a snow layer of thickness $d$, the internal $r$ and $t$, considering all multiple reflections, are given in Eqs. (24), (25), and (26):

$$
r = t_0 (1 - t_0^2) (1 - r d_0^2)^{-1},
$$
(24)

$$
t = r_0 (1 - r_0^2) (1 - t d_0^2)^{-1},
$$
(25)

where

$$
t_0 = \exp(-\gamma d')
$$
(26)

is the one-way transmissivity through the slab. The layer emissivity is obtained from $e = 1 - r - t$. Note that $e$, $r$, $t$ are to be indexed by the layer number $j$.

The way $\gamma_0$ and $\gamma_0'$ are related is determined by the internal scattering phase function and by the critical angle $\theta_c$ given by Eq. (11). The solid angle of the beams for $T_{0j}$ and $T_{2j}$ is

$$
\Omega_j = 2\pi \left(1 - \sqrt{\frac{n_j'^2 - 1}{n_j'}}\right)
$$
(27)

and the solid angle $\Omega_2$ of trapped radiation is the complement $2\pi - \Omega_1$; that is,

$$
\Omega_2 = 2\pi - \frac{\sqrt{n_j'^2 - 1}}{n_j'}.
$$
(28)

The total scattering coefficient $\gamma_j$ is the sum

$$
\gamma_j = 2\gamma_0 + 4\gamma_0',
$$
(29)

and, using Eqs. (27) and (28), we get for isotropy of scattering

$$
\frac{4\gamma_0}{\gamma_0} = \frac{\Omega_2}{\Omega_1}.
$$
(30)

With Eqs (27), (28), (29), and (30) we can determine the six-flux coefficients $\gamma_0$, $\gamma_0'$, $\gamma_0''$, and $\gamma_0''$, from $n_j$' and the two-flux coefficients $\gamma_0'$ and $\gamma_0''$, using Eqs. (17) and (18).

### Reflectivity at Layer Interfaces

The interface reflectivity $s_j$ between two regular layers is given by $s_j = |F_j|^2$, where $F_j$ is the Fresnel reflection coefficient of the interface between layers $j$ and $j+1$. By regular we mean that a simple transition occurs in the permittivities $\varepsilon_j$ to $\varepsilon_{j+1}$, and that coherent superpositions due to reflections at other interfaces can be ignored because if the layers are deep enough, the natural variability in thickness, local incidence angle, and also the different frequencies allowed within the radiometer bandwidth tend to cancel the interference effects. This is the usual situation. However, for layers thinner than about half a wavelength the variability is not large enough to cancel the coherent effects, and thus they cannot be ignored; in other words, the quarter-wavelength resonance must be taken into account. The one-way phase through layer $j$ is given by Eq. (31):

$$
P_j = \frac{2\pi d_j n_j' \cos \theta_{j-1}}{\lambda}
$$
(31)

which is a function of the thickness $d_j$ of layer $j$, the real part $n_j$ of the refractive index, the propagation angle $\theta_{j-1}$, and vacuum wavelength $\lambda$. Defining [Eq. (32)]

$$
E_j = e^{i\alpha j},
$$
(32)
the reflection coefficient $R_2$ of a coherent double layer (Fig. 4) can be written as Eq. (33),

$$R_2 = \frac{F_1 + F_2 e^{-F_2 t_2}}{1 + F_1 F_2}$$

and the reflectivity $s$ is given by Eq. (34),

$$s = |R_2|^2 = \frac{|F_1 + F_2 e^{-F_2 t_2}|^2}{|1 + F_1 F_2|^2}.$$  \hspace{1cm} (34)

For a lossless medium, we have Eq. (35):

$$s = \frac{F_1 + F_2}{1 + F_1 F_2} + 2 F_1 F_2 \cos(2P_2).$$

This is a valid approximation for radiation originating at the radiometer and propagating into the snowpack (i.e., we describe the reciprocal and equivalent path of photons as if they were emitted by the radiometer antenna).

**Effective Propagation Angle and Polarization Mixing**

So far it has been assumed that at any position within the snowpack the effective propagation angle $\theta_j$ (in layer $j+1$, see Fig. 1) with respect to the surface normal is given by Snell’s law for both vertically and horizontally polarized (up- and downwelling) radiation:

$$\sin \theta_j = \frac{\sin \theta_i}{n_{j+1}}.$$  \hspace{1cm} (37)

Here the index $s$ in $\theta_i$ refers to Snell. In addition to refraction, deviations in propagation direction occur due to volume scattering. The proposed change leads to a first-order correction in effective propagation path and polarization mixing for radiation originating at the radiometer and propagating into the snowpack (i.e., we describe the reciprocal and equivalent path of photons as if they were emitted by the radiometer antenna).

**Effective Propagation Angle**

For a radiometer observing at the incidence angle $\theta_i$, the effective propagation angle is well approximated by Eq. (37) as long as volume scattering is not too important. This is true just below the smooth snow surface. However, with increasing snow depth, more and more of the forward streaming flux consists of forward-scattered radiation, leading to an effective propagation angle which may differ from (37). Assuming that, in the limit of strong scattering, the forward (downward) and backward (upward) fluxes are diffuse, then the effective propagation angle $\theta_d$ can be represented by

$$\cos \theta_d = \frac{1 + \cos \theta_s}{2},$$  \hspace{1cm} (38)

where $\theta_s$ is the critical angle for total reflection, that is, the maximum propagation angle of radiation belonging to the vertical fluxes. It is given by Eq. (39):

$$\cos \theta_s = \sqrt{1 - (n_{j+1})^2}.$$  \hspace{1cm} (39)

For a dilute medium ($n_{j+1} = 1$), we get $\cos \theta_s = 0.5$; this is the usual value of the two-stream model Ishimaru (1978). In our case of six streams or “fluxes,” the streams have more narrow beams; therefore, we have to use the modified form given by Eq. (38).

The actual value of the effective propagation angle $\theta_j$ in layer $j+1$ is somewhere between $\theta_s$ and $\theta_d$. We assume that the two cosines are weighted according to the transmissivity of $t_i$ for nonscattered radiation from the snow surface (at height $z_s$) to height $z<z_s$, that is [Eq. (40)],

$$\cos \theta_j = t_s \cos \theta_s + (1-t_s) \cos \theta_d$$  \hspace{1cm} (40)

and $t_s$ is taken in Eq. (41) to be

$$t_s = \exp\left( -\int_{z_s}^{z_s} \frac{\gamma_z}{2 \cos \theta_{h'}} dz' \right).$$  \hspace{1cm} (41)

where $\gamma_z$ is the scattering coefficient at height $z'$. For negligible scattering $t_s \to 1$, leading to $\theta_j = \theta_s$, whereas for strong scattering we get $t_s \to 0$ and thus $\theta_j = \theta_d$.

**Polarization Mixing**

After diffuse scattering the photons lose their memory about where they came from. As a consequence, they cannot remember their plane of incidence which is needed to define the state of $h$ or $v$ polarization. Therefore, the $v$- and $h$-polarized fluxes are mixed in a similar way as $\theta_s$ and $\theta_d$. This mixing can be modeled by modifying the polarization-dependent parts of the snowpack model, that is, the reflectivities $s_{h}$ and $s_{v}$ at the interface between Layer $j$ and $j+1$. The computation of these reflectivities without polarization mixing is described in the second section. Here we describe the modification due to the mixing effect.

Let us denote the difference $\Delta s_j$ by
\[ \Delta f = s_{h} - s_{b}. \]  

This difference is reduced due to scattering, leading to effective interface reflectivities \( s_{\text{eff}} \) and \( s_{\text{eff}} \) between Layers \( j \) and \( j+1 \) at position \( z=z_{j} \):

\[ s_{\text{eff}} = 0.5[s_{h} + s_{b} + t \Delta f], \]  

\[ s_{\text{eff}} = 0.5[s_{h} + s_{b} - t \Delta f]. \]  

In Eqs. (43) and (44) the values of \( s_{h} \) and \( s_{b} \) are the ones computed for the angles \( \theta_{b} \). In a further improvement, these coefficients could be averaged over the range of incidence angles. However, except near the Brewster angle, the effects are small because they tend to cancel.

**Primary Model Parameters (Input Data)**

At a given frequency \( f \), polarization \( p \), and incidence angle \( \theta \), the following snow physical parameters are needed for each layer \( j (j=1, \ldots, n) \):

- density \( \rho_{j} \)
- temperature \( T_{j} \)
- liquid water content \( W_{j} \)
- correlation length \( p_{j} \)
- vertical position \( z_{j} \) or thickness \( d_{j} \)

Furthermore, the ground temperature \( T_{0} \) and the snow-ground reflectivity \( s_{g} \) are needed. These primary parameters define the derived secondary model parameters to be described below.

**Secondary Model Parameters**

**Dielectric Properties of Dry Snow**

For dry snow the real part of the microwave permittivity can be well expressed in Eqs. (45) and (46) according to Mätzler (1996) in terms of the density \( \rho \) in g/cm³:

\[ \varepsilon'_{d} = 1 + 1.5995\rho + 1.861\rho^{3}, \quad 0 < \rho < 0.4 \text{ g/cm}^{3}, \quad (45) \]

\[ \varepsilon''_{d} = ((1 - \nu)\nu_{s} + \nu_{e})^{3}, \quad \rho > 0.4 \text{ g/cm}^{3}, \quad (46) \]

where \( \varepsilon_{d} = 1.0, \varepsilon_{e} = 3.215 \) and \( \nu = \rho/0.917 \text{ g/cm}^{3} \).

The imaginary part of the dielectric constant of dry snow is given by Eq. (47):

\[ \varepsilon''_{d} = \sqrt{\varepsilon_{d}/\varepsilon_{e}}, \quad (47) \]

where \( K^{2} \) is the squared field factor (Fig. 1 in the companion article); for dry snow \( K^{2} \approx 0.44 \) for \( \nu = 0.33 \), then increasing to \( K^{2} = 0.54 \) at \( \nu = 0.5 \). A similar expression, optionally used by MEMLS, is the one proposed by Tiuri et al. (1984) [Eq. (48)]:

\[ \varepsilon''_{d} = \varepsilon''_{0}(0.52\rho + 0.62\rho^{3}). \quad (48) \]

For the imaginary part \( \varepsilon'' \) of the dielectric permittivity of ice, the formula of Mätzler (1999b) [see also Eqs. (61)–(64) of Wiesmann et al. (1998)] is used [Eqs. (49), (50), (51), and (52)]:

\[ \varepsilon'' = \frac{a}{f} + \beta f, \quad (49) \]

\[ \theta = \frac{300}{T} - 1, \quad (50) \]

\[ a = (0.00504 + 0.00062\theta) \exp(-22.1\theta), \quad (51) \]

\[ \beta = \frac{B_{1}}{T} \exp(b/T) \left( \exp(b/T) - 1 \right) + B f^{2} \]

\[ + \exp(-10.02 + 0.0364(T - 273) \text{ K}), \quad (52) \]

where \( f \) is the frequency in GHz, \( B_{1} = 0.0207 \text{ K GHz}^{1}, \) \( B = 335 \text{ K}, \) and \( B_{2} = 1.16 \times 10^{-11} \text{ GHz}^{2}. \)

**Dielectric Properties of Wet Snow**

Wet snow exhibits a distinct Debye relaxation spectrum in the microwave range. According to physical mixing models (e.g., Mätzler, 1987), for small values of the volumetric liquid water content \( W \), the effective permittivity \( \varepsilon = \varepsilon' + i\varepsilon'' \) is linear in \( W \) and given by four terms in Eq. (53):

\[ \varepsilon = \varepsilon_{d} + \varepsilon_{l} + \varepsilon_{w} + \varepsilon_{d}, \quad (53) \]

where \( \varepsilon_{d} \) is the permittivity of dry snow, and \( \varepsilon_{l}, \varepsilon_{w}, \varepsilon_{d} \) are Debye terms expressed in Eq. (54) as

\[ \varepsilon_{l} = \varepsilon_{l} + \frac{\varepsilon_{l} - \varepsilon_{l}}{1 - if f_{l}}, \quad k = a, b, c. \quad (54) \]

The Debye parameters \( \varepsilon_{l}, \varepsilon_{l}, f_{l} \) are given by the following three relations [Eqs. (55), (56), and (57)]

\[ \varepsilon_{l} = \frac{W}{1 + \varepsilon_{l} - 1}, \quad k = a, b, c, \quad (55) \]

\[ \varepsilon_{l} = \frac{W}{1 + \varepsilon_{l} - 1}, \quad k = a, b, c, \quad (56) \]

\[ f_{l} = \frac{f_{l}}{1 + \frac{\varepsilon_{l} - \varepsilon_{l}}{\varepsilon_{l} + \varepsilon_{l} - 1}}, \quad k = a, b, c, \quad (57) \]

where the static \( \varepsilon_{l} \) and infinite permittivity \( \varepsilon_{l} \) and the relaxation frequency \( f_{l} \) of liquid water at \( T = 0 \text{C} \) are: \( \varepsilon_{w} = 88, \varepsilon_{l} = 4.9, f_{l} = 9 \text{ GHz} \) (Ulaby et al., 1986). The depolarization factors are taken from experimental data: \( A_{1} = 0.005, A_{2} = 0.4975 \) (Mätzler, 1987).

**Absorption Coefficient**

The absorption coefficient \( \gamma_{a} \) follows from the imaginary part \( n'' \) of the refractive index, and thus from the complex permittivity, \( \varepsilon = \varepsilon' + i\varepsilon'' = (n' + in'')^{2} \), according to Eq. (58):

\[ \gamma_{a} = \frac{4\pi n''}{\lambda}, \quad (58) \]

where \( \lambda \) is the vacuum wavelength.
Scattering Coefficient

According to Wiesmann et al. (1998), the six-flux scattering coefficient can be obtained from snow physical properties. Several possible fits were found among which the following two were of equal quality with regard to the available snow-sample data:

\[
g_s = \left( \frac{9.2 \rho_s}{1 \text{ mm}} \right) \left( \frac{1.23 p}{1 \text{ g/cm}^3} \right) \left( \frac{0.54}{\text{cm}} \right) \left( \frac{0}{50 \text{ GHz}} \right)^{0.25}, \tag{59}
\]

\[
g_r = \left( \frac{3.16 \rho_s}{1 \text{ mm}} \right) + 295 \left( \frac{p_s}{1 \text{ mm}} \right) \left( \frac{0}{50 \text{ GHz}} \right)^{0.25}, \tag{60}
\]

where \(p_s\) is the exponential correlation length, that is, a length obtained by fitting the measured correlation function to \(\exp(-x/p_s)\). The range of validity includes snow densities from 0.1 g/cm\(^3\) to 0.4 g/cm\(^3\), \(p_s\) from 0.05 mm to 0.3 mm and frequencies from 5 GHz to 100 GHz. An extension to larger correlation lengths is given in the companion article (Mätzler and Wiesmann, 1999). For \(p_s<0.05\) mm, Eq. (60) should be used. In the following examples we will use \(g_s\) according to Eq. (59).

EXAMPLES

Emissivity of a Snowpack

MEMLS was written in Matlab 5 (MathWorks, 1996). All subroutines are commented, and a help text is available with help subroutine. Many simulations have been made so far, and the data seem in general to be realistic as long as the limited parameter ranges are respected. For illustration purposes we present an example of an emissivity simulation. Table 1 shows the input data, that is, the snow profile of 21 December 1995 found at Weissfluhjoch-Davos, Switzerland. It is a winter snow situation with a snow height of 60.3 cm. The top layer \((j=4)\) was 20 cm of new snow, below was a thin crust, and the bottom layer consisted of coarse-grained snow.

The measured and simulated emissivities as functions of frequency and incidence angle are shown in Figure 5. The lines indicate computed data while the symbols indicate measurements. Note the significant polarization difference at frequencies below 35 GHz, where Layer 3 acts as a coherent reflector in agreement with the measurements. The increase of the measured emissivity at small incidence angles is due to the shadowing of the snow cover by the radiometers. Whereas the emissivity at vertical polarization is slightly underestimated, the agreement is excellent at horizontal polarization. The broad minimum in the \(h\)-polarized spectrum around 25 GHz is the result of the 3 mm crust of Layer 3 acting there as a quarter-wave resonance. The transition from coherent to incoherent interaction takes place at 35 GHz where a kink is recognized. Table 2 shows the intermediate model data of the simulation at 25 GHz. The indices \(h\) and \(v\) stand for horizontal and vertical polarization, respectively. Layer 3 disappeared here because it is considered a coherent layer. Its effect is expressed by the large value of \(s_h\) for \(j=2\).

Simulation of Spatial Snowpack

Emissivity Variations

Sturm and Holmgren (1994) describe a vertical cross section through a snowpack covering a distance of several meters at Innavit Creek, Alaska, for 18 November 1989. Five layers were observed; they consisted of depth hoar and wind crusts in alternation. The situation was characterized by type, thickness of each layer, and soil temperature. The corresponding layer temperatures were interpolated from the soil and air temperatures. The correlation lengths and densities were specified using typical values from our snow sample catalog (Wiesmann et al., 1996). An extension to larger correlation lengths is given in the companion article (Mätzler and Wiesmann, 1999). For \(p_s<0.05\) mm, Eq. (60) should be used. In the following examples we will use \(g_s\) according to Eq. (59).

DISCUSSION AND OUTLOOK TO FURTHER DEVELOPMENT

The purpose of the present article was mainly to describe MEMLS and all its elements. First applications were used to show that reasonable results can be obtained. In order to further test the model and in order to assess its parameter range and errors, MEMLS should be applied to many kinds of snowpacks and firn with well-known physical and structural properties. Since the number of parameters can be large, it is difficult at present to give a perspective on the feasibility to retrieve special snow parameters from microwave measurements. However, this potential can now be studied by simulations. First sensitivity studies of MEMLS were done by...
Pulliainen et al. (1998). Some possible steps for future considerations are proposed in the following:

- Weighting functions, defined by $w_i = \partial c / \partial x_{ij}$, where $x_{ij}$ is the $i$th physical parameter of layer $j$, can be computed in order to assess the sensitivity of the model to any parameter.
- The assumption of an isotropic and exponential correlation function is not valid in general, but it is a reasonable simplification. Furthermore, it leads to a representative definition of the structure parameter, that is, the correlation length $p_{ex}$, which is obtained from fitting the observed correlation function to $\exp(-x/p_{ex})$ as described by Weise (1996). The index $e$ in $p_{ex}$ is meant to indicate this exponential fit. Wiesmann et al. (1998) used the slope of the correlation function at zero displacement $x=0$ to define a slightly different correlation length $p_c$. This leads to noisier and slightly different values, especially in case of fine-grained snow.
- By including the occasional snow-structure anisotropy (e.g., observed in new snow deposited under calm wind condition, surface hoar, depth hoar) the model can be improved, especially its polarization behavior at frequencies above 30 GHz.
- Occasionally, we observed clear deviations of the spatial correlation function from simple exponentials. Improvements can be expected by choosing more suitable functions. Using the improved Born approximation (Wätzler, 1998a) and measured correlation functions, it can be tested how these functions are related to snow types and how they influence the microwave properties.
- Further improvements of the polarization behavior of the model is possible by using a fully polarimetric radiative transfer code. However, the expected gain from this more elaborate approach is limited, because volume scattering in snow tends to be insensitive to polarization (Wiesmann et al., 1998). The most obvious polarization effects arise from reflections at layer interfaces which are treated by the present model.
- A limitation of MEMLS at very high frequencies

![Figure 5. Measured (circles, triangles) and simulated (curves) data of the snowpack of 21 December 1995. Left: Emissivity versus frequency at $\theta_a = 50^\circ$. Right: Emissivity versus incidence angle at $f = 21$ GHz.](image)

<table>
<thead>
<tr>
<th>$\theta$ (deg)</th>
<th>$d' (cm)$</th>
<th>$s_h$</th>
<th>$s_v$</th>
<th>$r$</th>
<th>$t$</th>
<th>$D_h (K)$</th>
<th>$D_v (K)$</th>
</tr>
</thead>
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<tr>
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<td>0.0535</td>
<td></td>
<td></td>
<td>252.9</td>
<td>255.9</td>
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<td>0.8358</td>
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</tr>
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<td>20.3</td>
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<tr>
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<td>0.0000</td>
<td>0.0150</td>
<td>0.9615</td>
<td>251.7</td>
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<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>253.4</td>
<td>251.7</td>
</tr>
</tbody>
</table>

$*$The upwelling brightness temperatures in each layer are given by $D_h$ and $D_v$. 

Table 2. Intermediate Model Data of the Simulation at 25 GHz and 50° Incidence Angle
Figure 6. Emissivity (upper lines) and snowheight (lower line) versus distance. Observation frequency is 37 GHz and incidence angle $\theta_i=50^\circ$. Input data are taken from Sturm and Holmgren (1994). The horizontal dashed line indicates the emissivity of the nearest SSM/I pixel at 37 GHz (v-pol.) assuming $T_{\text{sky}}=30$ K.

results from the assumption of smooth interfaces between layers and of a smooth snow surface. Improvements can be expected by including roughness effects.

- A limitation of the present MEMLS version is given by the limited range of correlation lengths. The range covers the snow structure observed at Weissfluhjoch during two winter seasons. Larger grains are observed for old wet snow; when it refreezes the scattering leads to extreme microwave signatures which cannot be accounted for by the present model. The companion article will be dedicated to this situation.

The complexity of MEMLS can be large. In fact, arguments were raised for preferring single-layer snow emission models as they may be more practical and thus more feasible. This statement can also be tested by simulations using MEMLS for different values of $n$, where the single-layer situation ($n=1$) is met as well. Nevertheless, it should be pointed out that layering is inherent to most snowpacks as described in the introduction and found in the examples.

REFERENCES


MathWorks, Inc. (1996), Matlab, the Language of Technical Computing, Natick, MA.


